AP Calculus BC

Estimating w/ Areas - Riemann Sums

1) $f(x) = -x^2$ on $[0,2]$

$$\text{Area} = \left| f(2)\cdot\tfrac{1}{2} + f(\tfrac{3}{2})\cdot\tfrac{1}{2} + f(1)\cdot\tfrac{1}{2} + f(\tfrac{1}{2})\cdot\tfrac{1}{2} \right|$$
$$= \left| \tfrac{1}{2}\left(-4 + -\tfrac{9}{4} + -1 + -\tfrac{1}{4}\right) \right| = \frac{15}{4}$$

Since $f'(x) < 0$ on $[0,2]$, $\frac{15}{4}$ is an overestimate of the area.

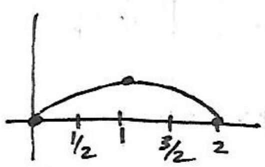
2) $f(x) = \cos x$ on $[0, \pi/2]$ $\quad \Delta x = \frac{\pi}{8}$

$$L_4 = \frac{\pi}{8}\left[\cos 0 + \cos\frac{\pi}{8} + \cos\frac{\pi}{4} + \cos\frac{3\pi}{8}\right]$$
$$= \frac{\pi}{8}\left[1 + \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} + \frac{\sqrt{2}}{2} + \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}\right]$$

* half-angle identity
$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

Since $f'(x) < 0$ on $[0, \frac{\pi}{2}]$, L_4 is is overestimate

3) $f(x) = 2x - x^2$ on $[0,2]$



a) $R_4 = \frac{1}{2}\left[f(2) + f(\tfrac{3}{2}) + f(1) + f(\tfrac{1}{2})\right]$
$= \frac{1}{2}\left[0 + 3 - \tfrac{9}{4} + 1 + 1 - \tfrac{1}{4}\right]$

b) $L_4 = \frac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right]$
$= \frac{1}{2}\left[0 + 1 - \tfrac{1}{4} + 1 + 3 - \tfrac{9}{4}\right]$

c) $M_2 = 1\left[f(\tfrac{1}{2}) + f(\tfrac{3}{2})\right]$
$= 1\left[1 - \tfrac{1}{4} + 3 - \tfrac{9}{4}\right]$

d) $T_2 = \frac{1}{2}\left[f(0) + 2f(1) + f(2)\right]$
$= \frac{1}{2}\left[0 + 2 + 0\right]$

4) a) $L_4 = 0.25\left[f(0) + f(0.25) + f(0.5) + f(0.75)\right]$

$0.25\left[1.0 + 0.8 + 1.3 + 1.1\right]$

b) $T_4 = \frac{1}{2}(0.25)\left[f(0) + 2f(0.25) + 2f(0.5) + 2f(0.75) + f(1.0)\right]$

$= \frac{1}{8}\left[1 + 1.6 + 2.6 + 2.2 + 3.2\right]$

c) $M_2 = 0.5\left[f(0.25) + f(0.75)\right]$

$= 0.5\left[0.8 + 1.1\right]$

5) $\int_0^{60} v(t)\,dt \approx L_6 = 10\left[v(0) + v(10) + v(20) + v(30) + v(40) + v(50)\right]$

$= 10\left[1 + 1.7 + 1.8 + 1.4 + 1.0 + 1.5\right]$

$\int_0^{60} v(t)\,dt$ is the total distance that the tide carries the bottle, in meters.